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## LEARNING AND PRODUCTION RATE IN COST ESTIMATING

Production rate is a very important factor in estimating manufacturing costs. Actual price data shows that production rate has a much greater affect on cost than learning. Cost analysts frequently use learning as the only variable in creating cost estimating relationships from historical databases for manufacturing costs. Predicting future costs due to changes in annual and cumulative production quantities should use both learning and production rate to prevent erroneous cost estimates. Large errors in predicting cost and quantities can result due to ignoring rate. High production rate lowers unit cost, and vice versa low production rate increases unit production cost. Lower production rate is a frequent reaction to cutting current costs, but this usually results in a higher unit production cost due to the government and contractor's fixed costs or business base. Production rate is shown to have a much greater impact than learning in manufacturing cost estimating relationships and unit production costs.

Alan G. Markell  
Operation Research Analyst  
U.S. Army, Missile Command  
Air-to-Ground Missile Systems Project Office  
SFAE-MSL-HD-M-E  
Redstone Arsenal, AL 35898-5610  
Comm (205) 876-9437  
DSN 746-9437

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# **Learning and Production Rate In Cost Estimating**

**Alan Markell  
Operations Research Analyst  
Program Executive Office, Tactical Missiles  
Air-to-Ground Missile Systems Project Office  
Redstone Arsenal, Alabama 35898-5610  
(205) 876-9437  
DSN 746-9437**

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**Learning and Production Rate in Cost Estimating**  
**by Alan G. Markell**

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**1. Introduction**

The cumulative average learning curve theory presumes that as the quantity doubles the cumulative average cost decreases by a constant percentage.

The unit learning curve theory presumes that as the quantity doubles the unit cost decreases by a constant percentage. Both of these theories are based upon observations of aircraft manufacturing plants.

In so far as it concerns Army weapon systems manufacturing cost, I think cost analysts should be using the following basic estimating equation

$$\text{COST} = \text{FUC} * ( \text{QC} ^ \text{B} ) * ( \text{QA} ^ \text{C} )$$

where

COST= average unit manufacturing cost

FUC = first unit cost

QC = cumulative production quantity

B = exponent of learning slope

QA = annual production quantity, or annual production rate

C = exponent of rate slope

This is a multi-variant cost estimating relationship with two independent variables, cumulative production quantity and annual production quantity, or in other words, production rate. Learning is related to cumulative production quantity. Rate is related to annual production rate.

## Learning and Production Rate in Cost Estimating

People that work with contracts and real data have a feeling that production rate is an important cost factor. For instance, if a production program gets annual quantities reduced, the program management staff expects an increase in unit price although a cost estimating relationship using learning only, that is, without a rate factor, would not show an increase in price.

The Selected Acquisition Report addresses rate with the reporting of Maximum Economic Rate for production in section 17, Production Rate Data. DOD 5000.2-M, Department of Defense Manual, Defense Acquisition Management Documentation and Reports, Part 17, Attachment 1 and Attachment 2, Selected Acquisition Report Preparation Instructions, February 1991 gives the following definition:

**Maximum Economic Production Rate** is defined as the production rate at which the lowest unit cost is attainable with the facilities and tooling currently programed to be available.

The SAR Handbook addresses rate with the following definition:

**Minimum Sustaining Production Rate** is defined as the production rate necessary to keep production lines open while maintaining a responsive vendor/supplier base. Any reduction in production below the minimum sustaining rate causes a dramatic rise in unit cost.

Cost analysts that develop cost estimating relationships with historical data usually assume a single independent variable, cumulative production, and disregard production rate, that is, learning is the only factor that affects cost; rate has no affect. This is a convenient and simplistic assumption. Their resulting cost estimating relationship is like the blind man's description of an elephant; if he grabs the elephant's tail he says an elephant is like a rope; if he grabs the elephants leg, he says an elephant is like a tree. This is after-the-fact cost estimating.

For cost analysts that must estimate future costs, and the important word is future, using cumulative production quantity or learning without consideration for annual production rate leads to erroneous cost estimates. This is before-the-fact cost estimating.

## Learning and Production Rate in Cost Estimating

With the information I am about to show you, you will see that annual production rate has a much greater affect on cost than cumulative production quantity or learning. You will see that cumulative production quantity or learning is a relatively minor factor.

### 2. Actual Data

Actual cost data submitted to Air-to-Ground Missile Systems Project Office will illustrate the importance of production rate. For each fiscal year, the contractor provided to AGMS Project Office a yearly cumulative average unit price in then-year dollars for each quantity of missiles in each fiscal year, similar to as show in figure 1. This cost data is taken from Contract DAAH01-90-C-0323, Modification PZ0004, Attachments 10, 11, 12, and 13.

### 3. Data Manipulation

I took the price data which was submitted to Air-to-Ground Missile Systems Project Office in escalated dollars and converted them to FY93 constant dollars to prepare the data for log-linear regression. Next I take the logarithm of these constant dollars values and annual production quantities. I used Lotus 1-2-3 to perform single variant log-linear regression. The independent variable is annual cumulative production quantity, X. The dependent variable, Y, is the cumulative average unit cost with respect to one annual lot. These regressions gives me the first unit cost and slope for each year individually. The graph of Hellfire II missiles unit price verses quantity for the individual years are shown on the figure 2.

To quantify the affects of two independent variables, learning and production rate, multi-variant log-linear regression was performed on the price data. The independent variables are cumulative production quantity, QC, and annual production rate, QA. The dependent variable, Y, is the cumulative average unit price.

For the regression, I normalized the production rate quantity, QA. In the case of Hellfire II, I normalized QA such that 6,300 units in a year equals 1. For instance, 2,100 normalized equals 0.33. The quantity of 6,300 is the Maximum Economic Rate. The reason for normalizing QC, is that in the equation  $Y = FUC * [ QC ^ B ] * [ QA ^ C ]$ , the production rate term,  $QA^C$ , equals 1 and has no effect when Maximum Economic Rate occurs; only learning has an effect.

## Learning and Production Rate in Cost Estimating

Intuitive, this makes good sense to me, but it is not totally necessary. The predicted cost is exactly the same value irregardless of whether the production rate factor is normalized or not.

I used Lotus 1-2-3 to perform multi-variant log-linear regression. The actual and regression average unit price verses quantity of Hellfire II missiles are shown on figure 3.

### 4. Analysis

Figure 4 shows the coefficients due to learning, rate, and the product of learning and rate. The graph in figure 4 clearly shows rate is the dominate factor. Learning is almost constant irregardless of cumulative production quantity.

The values of first unit cost and slope for each year and the combined years are shown on figure 5 for Hellfire II Missile.

Hellfire II missile cost data shows the learning factor has a 98% slope and production rate factor has an 85% slope.

A Slope value of 98% shows that Learning is an insignificant factor in cost. The production rate factor has an overwhelmingly more important affect on cost than learning.

Using production rate as a cost factor provides a plausible, logical explanation and justification for cost estimates. Using a learning factor, without regard to rate, sometimes gives ridiculous cost estimates.

In actual practice, Air-to-Ground Missile Systems Project Office uses a separate cost estimating equation for each individual year rather than one equation for all years.

## 5. Examples

### Example 1 - Stretched-out Production Program - Cost Impact

The following is an example of how a stretched-out production program impacts cost. In this example, cumulative production quantity remains the same, but the production rate is reduced by 50% from the original plan. Intuitively, stretching-out a production program should increase total costs, but using learning factor only, and disregarding production rate, we will have a 2% increase in cost in a stretched-out program over the original compressed production schedule. This 2% is due to inflation. For example figure 6 shows cost with learning only and it shows the same program with learning and production rate as a factor. The cost are very different between the two different scenarios. The "learning and rate" cost estimate is 21% more for the same quantity, but slower production rate. By using "learning only" we would have under-estimated cost by 16%. The cost estimate where rate is a factor gives a more accurate and plausible cost estimate. To perform better cost estimates, we need to show the affect of production rate. Production rate substantial below maximum economic rate, increases unit production costs.

### Example 2 - low coefficient of determination, $R^2$

In the following example, the coefficient of determination,  $R^2$ , is shown to have a low value due to not taking production rate into account. The coefficient of determination,  $R^2$ , is a measure of the closeness of fit of the prediction equation to the actual data. In the cost world, a low  $R^2$  means our prediction equation is unreliable, and there is a weak relationship between quantity and price.

The regression of the maximum quantities in all years gives an  $R^2$  value of .63.

The regression of minimum quantities in all years gives an  $R^2$  value of .2.

We have an  $R^2$  of .96 using the same points and having prior knowledge of the affect of rate and learning.

Figure 7 shows these curves.



## Learning and Production Rate in Cost Estimating

### 6. Conclusion

For manufacturing costs, we should be using the equation  $COST = FUC * (QC^B)(QA^C)$  where QC equals cumulative quantity and QA equals annual production quantity. Or in other words,  $QC^B$  is the learning factor and  $QA^C$  is the rate factor.

New major weapon systems should have rate as a factor in estimating future manufacturing costs. A means to determine the rate and learning factors is to have this a part of the Design-To-Cost Program.

Production rate at maximum economic rate decreases unit production costs.

Producing the same quantity over more years will increase unit costs.

A decrease in production rate should increase unit production price, but using "learning factor only", we would NOT have an increase in unit production costs, which is wrong.

Cumulative production quantity, or experience, does not cause cost reductions, but rather cumulative production quantity (experience) provides an opportunity to reduce costs by alerting management of opportunities to reduce cost. Left unmanaged, costs increase. Cost reductions are due to a concerted effort to lower costs.

I would like to mention that using learning and rate factors are available on the new Department of Defense Baseline Cost Model ACEIT. They are also available on MICOM's Pices.

## Learning and Production Rate in Cost Estimating

### 7. References

HELLFIRE Optimized Missile System (HOMS)  
Martin Marietta Corporation  
Contract DAAH01-90-C-0323, Modification PZ0004  
Attachments 10, 11, 12, and 13  
Prices for Production Options  
dated December 1989

DOD 5000.2-M, Department of Defense Manual, Defense  
Acquisition Management Documentation and Reports, Part 17,  
Attachment 1 and Attachment 2, Selected Acquisition Report  
Preparation Instructions, February 1991

Selected Acquisition Report Handbook

# **Learning and Production Rate in Cost Estimating**

## **FIGURES**

First Fiscal Year Procurement

Quantity	Price (TY\$)
1,300	\$32,903
1,501	\$32,900
1,502	\$32,896
1,503	\$32,893
.	.
5,000	\$27,400

Second Fiscal Year Procurement

Quantity	Price (TY\$)
2,000	\$28,780
2,001	\$28,778
2,002	\$28,776
2,003	\$28,773
.	.
6,000	\$24,407

Third Fiscal Year Procurement

Quantity	Price (TY\$)
2,500	\$27,832
2,501	\$27,831
2,502	\$27,829
2,503	\$27,827
.	.
6,000	\$24,407

figure 1

# Hellfire II Price

(FY93c\$)

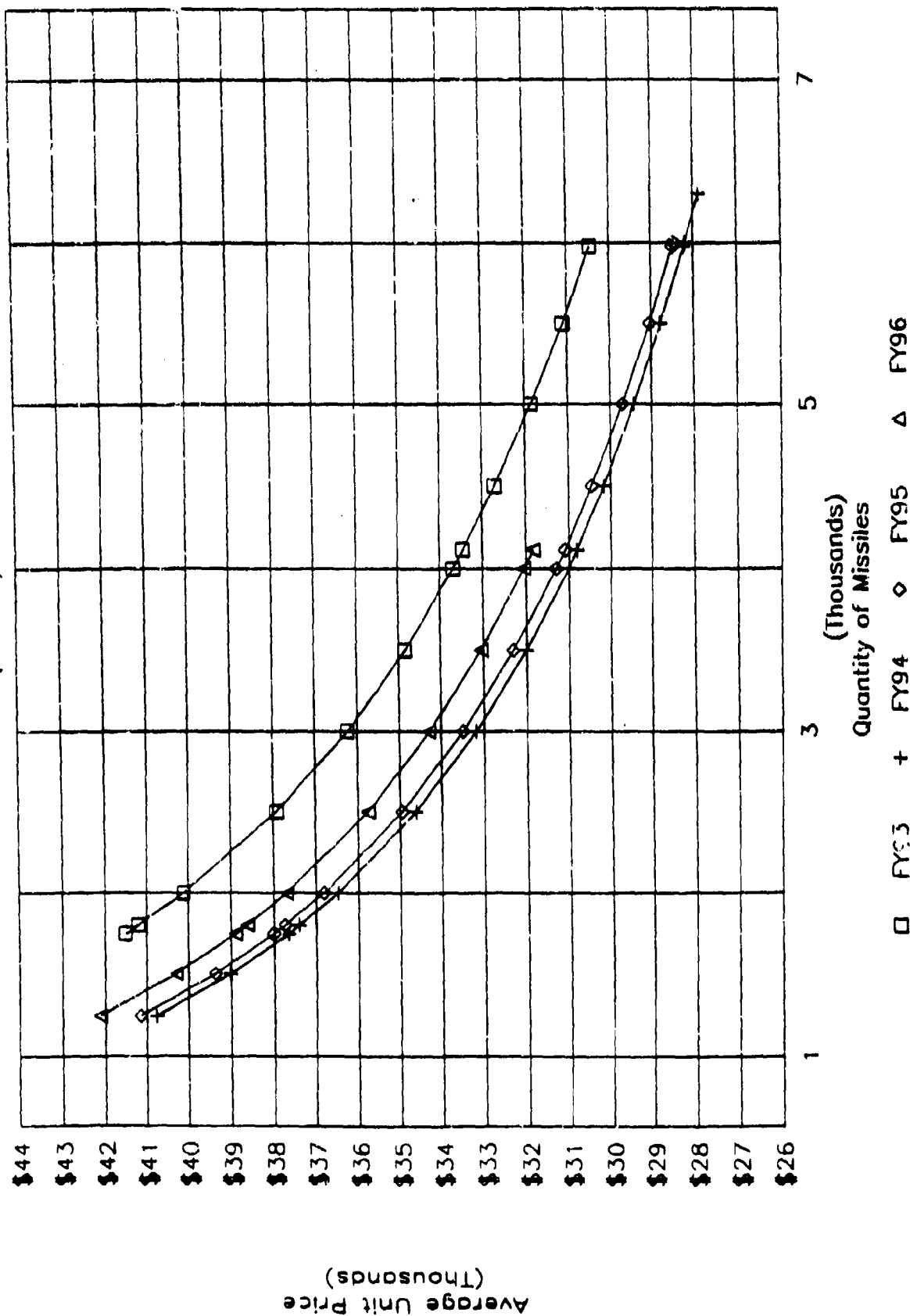


figure 2

# Hellfire II Missile Price

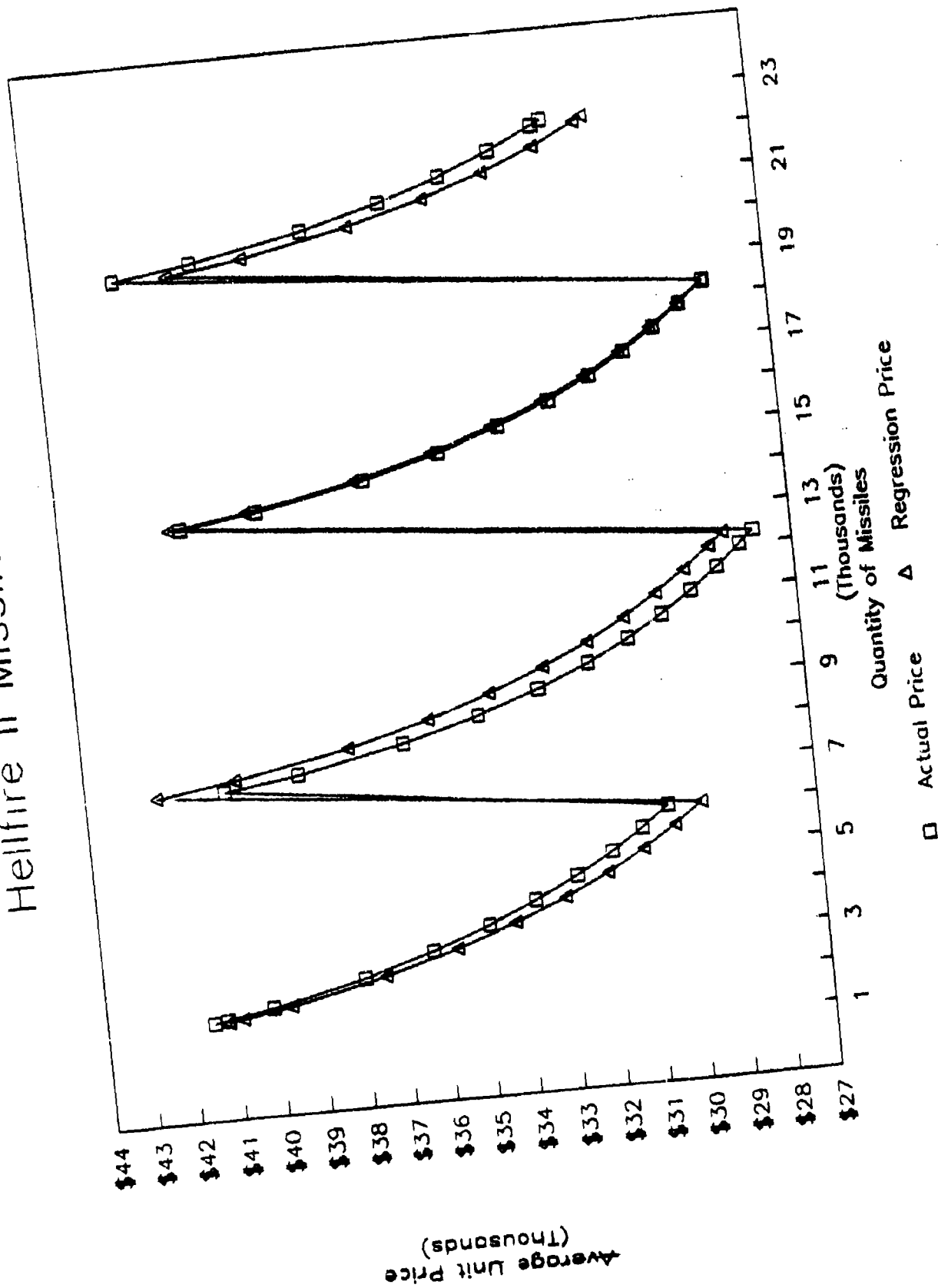


figure 3

# Hellfire II

Learning and Rate Factors

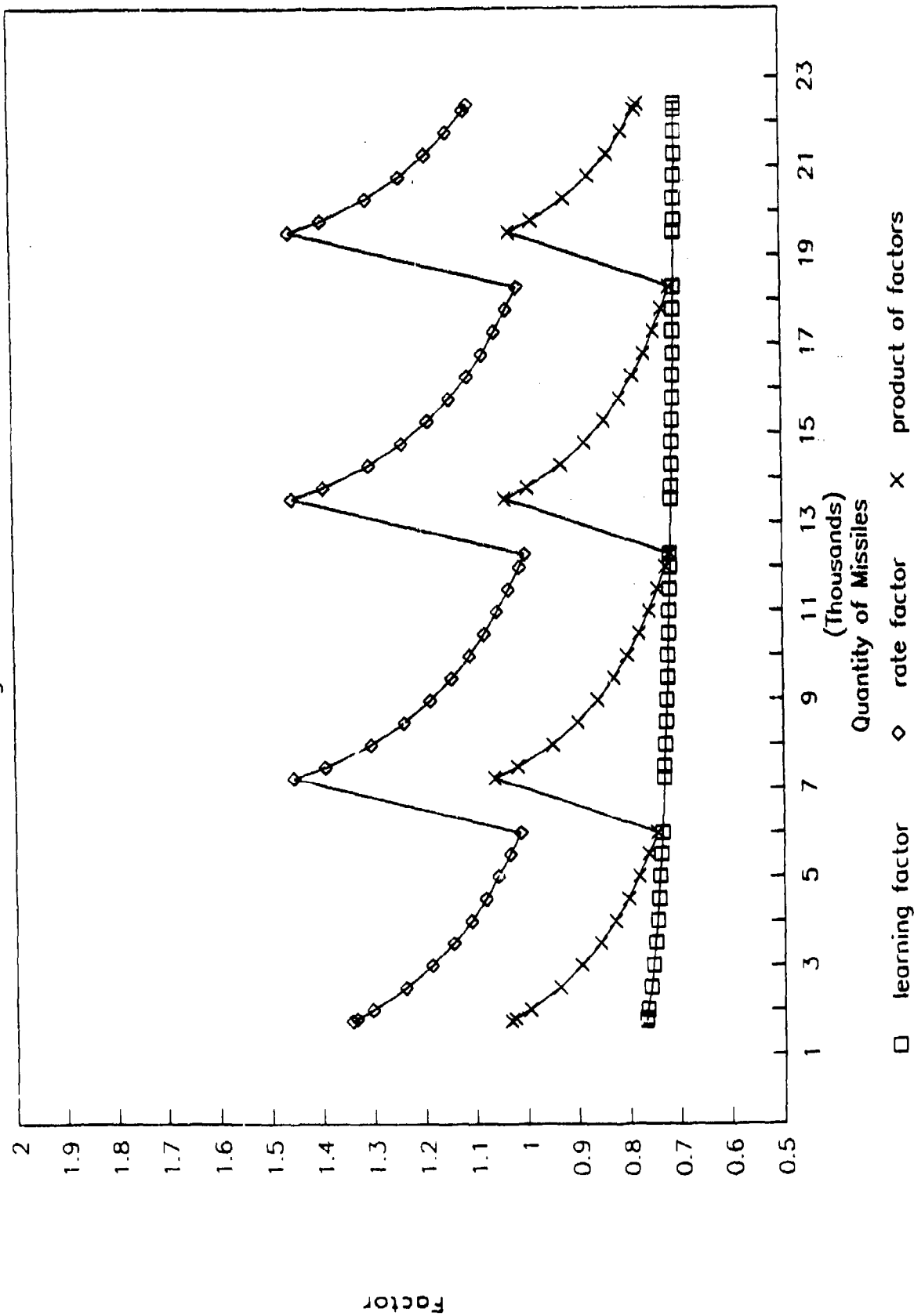


figure 4A

# HELLFIRE II

Learning and Rate Factor

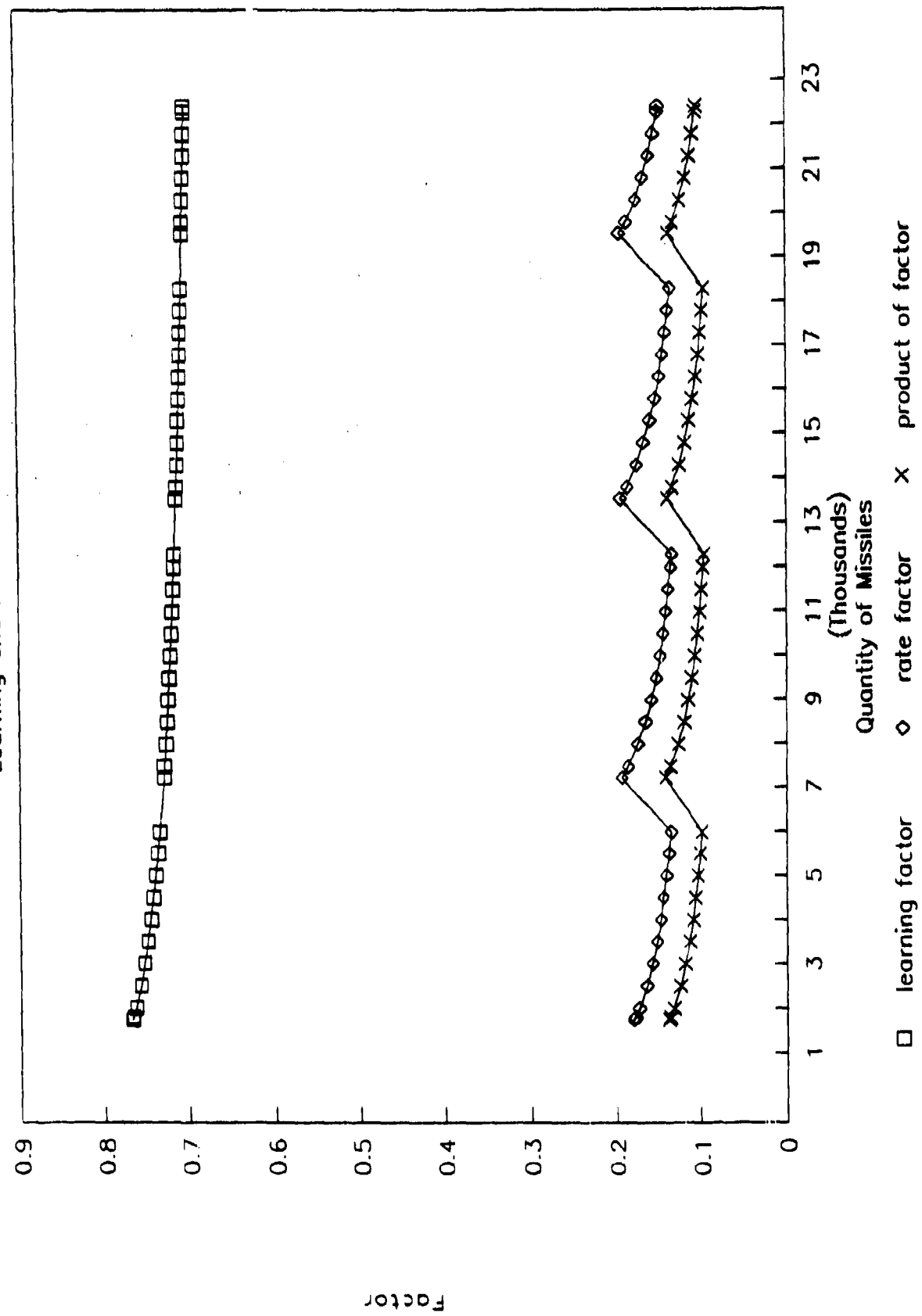


figure 4B



# HELLFIRE II Price

year	FUC (FY93c\$)	learning slope	rate slope
FY93	\$271,949		84.0%
FY94	\$216,802		85.0%
FY95	\$218,817		85.0%
FY96	\$223,971		85.0%
all years	\$300,594	97.6%	85.2%

figure 5

# Procurement Scenarios

ORIGINAL PLAN	FY93	FY94	FY95	FY96	TOTAL
annual quantity	4,000	4,000	0	0	8,000
cost (mFY93c\$)	\$132.1	\$128.9	\$0	\$0	\$261.0
cost (mTY\$)	\$137.9	\$137.5	\$0	\$0	\$275.5

## LEARNING FACTOR ONLY (without Rate Factor)

$$\text{cost} = \$38,218 * (\text{QC} ^{-0.0176}) * \text{QC}$$

REDUCE RATE BY 50%						Change	
annual quantity	2,000	2,000	2,000	2,000	8,000	-50%	production rate
cumulative quantity	2,000	4,000	6,000	8,000	8,000	0%	total quantity
cost (mFY93c\$)	\$66.9	\$65.2	\$64.6	\$64.3	\$261.0	0%	cost (mFY93c\$)
cost (mTY\$)	\$69.8	\$69.6	\$70.6	\$71.6	\$281.6	2%	cost (mTY\$)

"Reduce Rate by 50%" scenario causes a 2% increase in cost.

## LEARNING AND RATE FACTORS

$$\text{cost} = \$300,594 * (\text{QC} ^{-0.035395}) * (\text{QA} ^{-0.230860}) * \text{QA}$$

REDUCE RATE BY 50%						Change	
annual quantity	2,000	2,000	2,000	2,000	8,000	-50%	production rate
cumulative quantity	2,000	4,000	6,000	8,000	8,000	0%	total quantity
cost (mFY93c\$)	\$79.5	\$77.5	\$76.4	\$75.6	\$309.0	18%	cost (mFY93c\$)
cost (mTY\$)	\$82.9	\$82.7	\$83.4	\$84.3	\$333.4	21%	cost (mTY\$)

"Reduce Rate by 50%" scenario causes a cost increase of \$57.9 million or 21%.

Comparing Learning and Rate Scenario to Learning Only Scenario

"Reduce Rate by 50%" scenario causes an under-estimation of cost of \$51.8 million or 16%.

figure 6

# Hellfire II Missile Price

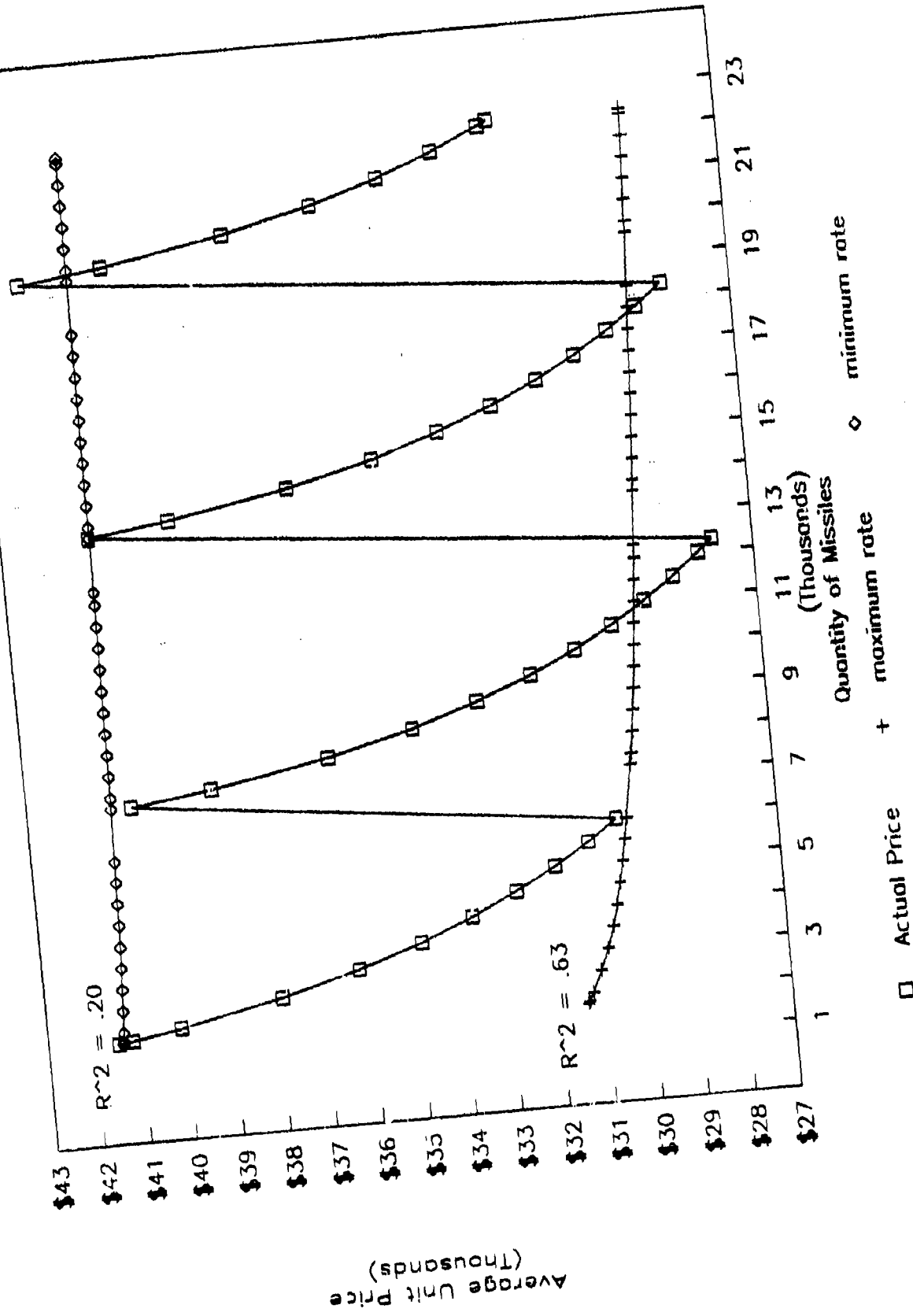


figure 7

**Learning and Production Rate  
in Cost Estimating**

**APPENDIX**

## HELLFIRE II in FY93c\$ for FY93 procurement

FY93esc\$ to FY93c\$ conversion factor = 0.957854 = 1 / 1.0440

X		Y						
Quantity	Propose	Propose	Regress	delta		log(X)	log(Y)	Regress
	(FY93esc\$)	(FY93c\$)	(FY93c\$)					log(Y)
1			\$271,949		*	0.00000		5.43449
1,747	\$43,340	\$41,613	\$41,613	\$0.17	*	3.24229	4.61819	4.61819
1,800	\$43,015	\$41,202	\$41,202	\$0.06	*	3.25527	4.61492	4.61492
2,000	\$41,889	\$40,124	\$40,123	\$0.09	*	3.30103	4.60340	4.60340
2,500	\$39,600	\$37,931	\$37,931	(\$0.45)	*	3.39794	4.57899	4.57900
3,000	\$37,824	\$36,230	\$36,230	\$0.19	*	3.47712	4.55907	4.55906
3,500	\$36,384	\$34,851	\$34,851	\$0.02	*	3.54407	4.54221	4.54221
4,000	\$35,181	\$33,698	\$33,698	(\$0.12)	*	3.60206	4.52761	4.52761
4,500	\$34,153	\$32,714	\$32,714	(\$0.18)	*	3.65321	4.51473	4.51473
5,000	\$33,259	\$31,857	\$31,857	(\$0.14)	*	3.69897	4.50321	4.50321
5,500	\$32,471	\$31,102	\$31,102	\$0.42	*	3.74036	4.49280	4.49279
5,976	\$31,799	\$30,459	\$30,459	(\$0.05)	*	3.77641	4.48371	4.48371

## Regression Output:

Constant 5.4344875 \$271,949  
 Std Err of Y Est 0.0000030  
 R Squared 0.9999999  
 No. of Observations 11  
 Degrees of Freedom 9

X Coefficient(s) -0.251766 slope = 83.99%  
 Std Err of Coef. 0.0000049

Y = \$271,949 \* X ^ b  
 \$30,459 = \$271,949 \* 5,976 ^ -0.25176

# HELLFIRE II in FY93c\$ for FY94 procurement

FY94esc\$ to FY93c\$ conversion factor = 0.937294 = 1 / 1.0669

X	Y							
Quantity	Propose	Propose	Regress	delta		log(X)	log(Y)	Regress
(FY94esc\$)	(FY93c\$)	(FY93c\$)	(FY93c\$)					log(Y)
1			\$216,802		*	0.00000		5.33606
1,245	\$43,498	\$40,770	\$40,771	(\$0.29)	*	3.09817	4.61035	4.61035
1,500	\$41,639	\$39,028	\$39,028	\$0.15	*	3.17809	4.59138	4.59138
2,000	\$38,923	\$36,482	\$36,482	\$0.12	*	3.30103	4.56208	4.56208
2,500	\$36,939	\$34,623	\$34,623	\$0.19	*	3.39794	4.53936	4.53936
3,000	\$35,393	\$33,174	\$33,174	(\$0.00)	*	3.47712	4.52079	4.52079
3,500	\$34,137	\$31,998	\$31,998	\$0.35	*	3.54407	4.50510	4.50510
4,000	\$33,084	\$31,009	\$31,010	(\$0.39)	*	3.60208	4.49149	4.49150
4,500	\$32,183	\$30,165	\$30,165	(\$0.23)	*	3.65321	4.47950	4.47951
5,000	\$31,398	\$29,429	\$29,429	\$0.08	*	3.69897	4.46878	4.46878
5,500	\$30,704	\$28,779	\$28,779	(\$0.07)	*	3.74036	4.45907	4.45907
6,000	\$30,084	\$28,198	\$28,198	(\$0.02)	*	3.77815	4.45021	4.45021
6,300	\$29,742	\$27,877	\$27,877	\$0.16	*	3.79934	4.44525	4.44524

## Regression Output:

Constant 5.3360629 \$216,802  
Std Err of Y Est 0.0000029  
R Squared 0.9999999  
No. of Observations 12  
Degrees of Freedom 10

X Coefficient(s) -0.234466 slope = 85.00%  
Std Err of Coef. 0.0000038

Y = \$216,802 \* X ^ b  
\$27,877 = \$216,802 \* 6,300 ^ -0.23446

# HELLFIRE II in FY93c\$ for FY95 procurement

FY95esc\$ to FY93c\$ conversion factor = 0.916758 = 1 / 1.0908

X		Y							
Quantity	Propose	Propose	Regress	delta		log(X)	log(Y)	Regress	
(FY95esc\$)	(FY93c\$)	(FY93c\$)	(FY93c\$)					log(Y)	
1			\$218,817		*	0.00000		5.34008	
1,245	\$44,886	\$41,150	\$41,149	\$0.23	*	3.09517	4.61437	4.61436	
1,500	\$42,967	\$39,390	\$39,390	\$0.02	*	3.17609	4.59539	4.59539	
2,000	\$40,164	\$36,821	\$36,821	(\$0.32)	*	3.30103	4.66609	4.56610	
2,500	\$38,117	\$34,944	\$34,944	\$0.02	*	3.39794	4.54337	4.54337	
3,000	\$36,522	\$33,482	\$33,482	\$0.12	*	3.47712	4.52481	4.52481	
3,500	\$35,225	\$32,293	\$32,293	(\$0.38)	*	3.54407	4.50911	4.50911	
4,000	\$34,140	\$31,298	\$31,298	\$0.33	*	3.60206	4.49552	4.49551	
4,500	\$33,210	\$30,446	\$30,446	\$0.25	*	3.65321	4.48352	4.48352	
5,000	\$32,399	\$29,702	\$29,702	(\$0.34)	*	3.69897	4.47279	4.47279	
5,500	\$31,683	\$29,046	\$29,046	(\$0.34)	*	3.74036	4.46308	4.46309	
6,000	\$31,044	\$28,460	\$28,459	\$0.42	*	3.77615	4.45423	4.45423	

## Regression Output:

Constant 5.3400807 \$218,817  
Std Err of Y Est 0.0000044  
R Squared 0.9999999  
No. of Observations 11  
Degrees of Freedom 9

X Coefficient(s) -0.234467 slope = 85.00%  
Std Err of Coef. 0.0000060

Y = \$218,817 \* X ^ b  
\$28,459 = \$218,817 \* 6,000 ^ -0.23446

# HELLFIRE II in FY93c\$ for FY96 procurement

FY96esc\$ to FY93c\$ conversion factor = 0.896941 = 1 / 1.1149

X	Y							
Quantity	Propose	Propose	Regress	delta		log(X)	log(Y)	Regress
	(FY96esc\$)	(FY93c\$)	(FY93c\$)					log(Y)
1			\$223,971		*	0.00000		5.35019
1,245	\$46,959	\$42,119	\$42,120	(\$0.23)	*	3.09517	4.62448	4.82449
1,500	\$44,952	\$40,319	\$40,319	\$0.10	*	3.17609	4.60551	4.60551
2,000	\$42,020	\$37,689	\$37,689	\$0.16	*	3.30103	4.57622	4.57622
2,500	\$39,876	\$35,768	\$35,768	\$0.09	*	3.39794	4.55350	4.55350
3,000	\$36,209	\$34,271	\$34,271	(\$0.12)	*	3.47712	4.53493	4.53493
3,500	\$36,853	\$33,055	\$33,055	\$0.18	*	3.54407	4.51924	4.51923
4,000	\$36,717	\$32,036	\$32,036	\$0.11	*	3.60206	4.50564	4.50564
4,114	\$35,482	\$31,825	\$31,825	(\$0.29)	*	3.61426	4.50277	4.50278

## Regression Output:

Constant 5.3501920 \$223,971  
Std Err of Y Est 0.0000024  
R Squared 0.9999999  
No. of Observations 8  
Degrees of Freedom 6

X Coefficient(s) -0.234464 slope = 85.00%  
Std Err of Coef. 0.0000047

Y = \$223,971 \* X ^ b  
\$31,825 = \$223,971 \* 4,114 ^ -0.23446



## HELLFIRE II MULTI-VARIANT REGRESSION

## Regression Output:

Constant 5.477980 \$300,594  
 Std Err of Y Est 0.010587  
 R Squared 0.962173  
 No. of Observations 42  
 Degrees of Freedom 39  
 Learning Rate  
 X Coefficient(s) -0.035395 -0.23086  
 Std Err of Coef. 0.0051563 0.007788  
 slope = 97.584 85.214

$$\text{COST} = \$300,594 * ( \text{QC} ^{-0.035395} ) * ( \text{QA} ^{-0.23086} )$$

QC	QA	Y									*log(QC)log(QA)	log(Y)
Qty	Qty	Proposal	Proposal	regression	learn	rate	factors	Propose	delta	*	*	
Cumulative	annual	Price	Price	(FY93c\$)	factor	factor	product	vs		*	*	
		(TY\$)	(FY93c\$)					Regress		*	*	
1	1			\$300,594	1.00	1.00	1.00			*	0.000	
1,747	1,747	\$43,340	\$41,513	\$41,180	0.77	0.18	0.14	0.8%	\$333	* 3.242	3.242	4.618
1,800	1,800	\$43,015	\$41,202	\$40,854	0.77	0.18	0.14	0.8%	\$348	* 3.255	3.255	4.615
2,000	2,000	\$41,889	\$40,124	\$39,724	0.76	0.17	0.13	1.0%	\$400	* 3.301	3.301	4.603
2,500	2,500	\$39,600	\$37,931	\$37,433	0.76	0.16	0.12	1.3%	\$499	* 3.398	3.398	4.579
3,000	3,000	\$37,824	\$36,236	\$35,659	0.75	0.16	0.12	1.6%	\$571	* 3.477	3.477	4.559
3,500	3,500	\$36,384	\$34,851	\$34,225	0.75	0.15	0.11	1.8%	\$626	* 3.544	3.544	4.542
4,000	4,000	\$35,181	\$33,698	\$33,029	0.75	0.15	0.11	2.0%	\$669	* 3.602	3.602	4.528
4,500	4,500	\$34,153	\$32,714	\$32,010	0.74	0.14	0.11	2.2%	\$704	* 3.653	3.653	4.515
5,000	5,000	\$33,259	\$31,857	\$31,124	0.74	0.14	0.10	2.3%	\$733	* 3.699	3.699	4.503
5,500	5,500	\$32,471	\$31,102	\$30,344	0.74	0.14	0.10	2.4%	\$758	* 3.740	3.740	4.493
5,976	5,976	\$31,799	\$30,458	\$29,681	0.74	0.13	0.10	2.6%	\$778	* 3.776	3.776	4.484
7,221	1,245	\$43,498	\$40,770	\$42,349	0.73	0.19	0.14	-3.9%(\$1,578)*	3.859	3.095	4.610	
7,476	1,500	\$41,639	\$39,028	\$40,516	0.73	0.18	0.13	-3.8%(\$1,488)*	3.874	3.176	4.591	
7,976	2,000	\$38,923	\$36,482	\$37,826	0.73	0.17	0.13	-3.7%(\$1,343)*	3.902	3.301	4.562	
8,476	2,500	\$36,939	\$34,623	\$35,849	0.73	0.16	0.12	-3.5%(\$1,227)*	3.928	3.398	4.539	
8,976	3,000	\$35,393	\$33,174	\$34,302	0.72	0.16	0.11	-3.4%(\$1,128)*	3.953	3.477	4.521	
9,476	3,500	\$34,137	\$31,996	\$33,039	0.72	0.15	0.11	-3.3%(\$1,043)*	3.977	3.544	4.505	
9,976	4,000	\$33,084	\$31,009	\$31,978	0.72	0.15	0.11	-3.1% (\$969)*	3.999	3.602	4.491	
10,476	4,500	\$32,183	\$30,165	\$31,066	0.72	0.14	0.10	-3.0% (\$901)*	4.020	3.653	4.480	
10,976	5,000	\$31,398	\$29,429	\$30,270	0.72	0.14	0.10	-2.9% (\$841)*	4.040	3.699	4.469	
11,476	5,500	\$30,704	\$28,779	\$29,564	0.72	0.14	0.10	-2.7% (\$786)*	4.060	3.740	4.459	
11,976	6,000	\$30,084	\$28,198	\$28,933	0.72	0.13	0.10	-2.6% (\$735)*	4.078	3.778	4.450	

12,276	6,300	\$29,742	\$27,877	\$28,584	0.72	0.13	0.10	-2.5%	(\$707)*	4.089	3.799	4.445
13,521	1,245	\$44,886	\$41,150	\$41,419	0.71	0.19	0.14	-0.7%	(\$269)*	4.131	3.095	4.614
13,776	1,500	\$42,957	\$39,390	\$39,649	0.71	0.18	0.13	-0.7%	(\$259)*	4.139	3.176	4.595
14,276	2,000	\$40,164	\$36,821	\$37,054	0.71	0.17	0.12	-0.6%	(\$234)*	4.155	3.301	4.566
14,776	2,500	\$38,117	\$34,944	\$35,151	0.71	0.16	0.12	-0.6%	(\$207)*	4.170	3.398	4.543
15,276	3,000	\$36,522	\$33,482	\$33,662	0.71	0.16	0.11	-0.5%	(\$181)*	4.184	3.477	4.525
15,776	3,500	\$35,225	\$32,293	\$32,449	0.71	0.15	0.11	-0.5%	(\$156)*	4.198	3.544	4.509
16,276	4,000	\$34,140	\$31,298	\$31,429	0.71	0.15	0.10	-0.4%	(\$131)*	4.212	3.602	4.496
16,776	4,500	\$33,210	\$30,446	\$30,553	0.71	0.14	0.10	-0.4%	(\$107)*	4.225	3.653	4.484
17,276	5,000	\$32,399	\$29,702	\$29,788	0.71	0.14	0.10	-0.3%	(\$86)*	4.237	3.699	4.473
17,776	5,500	\$31,683	\$29,046	\$29,110	0.71	0.14	0.10	-0.2%	(\$64)*	4.250	3.740	4.463
18,276	6,000	\$31,044	\$28,460	\$28,503	0.71	0.13	0.09	-0.2%	(\$43)*	4.262	3.778	4.454
19,521	1,245	\$46,959	\$42,119	\$40,884	0.70	0.19	0.14	2.9%	\$1,235 *	4.291	3.095	4.624
19,776	1,500	\$44,952	\$40,319	\$39,145	0.70	0.18	0.13	2.9%	\$1,175 *	4.296	3.176	4.606
20,276	2,000	\$42,020	\$37,689	\$36,597	0.70	0.17	0.12	2.9%	\$1,092 *	4.307	3.301	4.576
20,776	2,500	\$39,878	\$35,768	\$34,730	0.70	0.16	0.12	2.9%	\$1,039 *	4.318	3.398	4.553
21,276	3,000	\$38,209	\$34,271	\$33,270	0.70	0.16	0.11	2.9%	\$1,001 *	4.328	3.477	4.535
21,776	3,500	\$36,853	\$33,055	\$32,080	0.70	0.15	0.11	2.9%	\$975 *	4.338	3.544	4.519
22,276	4,000	\$35,717	\$32,036	\$31,082	0.70	0.15	0.10	3.0%	\$954 *	4.348	3.602	4.506
22,390	4,114	\$35,482	\$31,825	\$30,875	0.70	0.15	0.10	3.0%	\$950 *	4.350	3.614	4.503

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### Regression Output:

Constant	4.600850	\$39,889
Std Err of Y Est	0.010587	
R Squared	0.962173	
No. of Observations	42	
Degrees of Freedom	39	
	Learning Rate	
X Coefficient(s)	-0.0363	-0.23086
Std Err of Coef.	0.00515	0.007788
slope =	97.58%	85.21%

$$\text{COST} = \$39,889 * ( \text{QC} ^{-0.035395} ) * ( \text{QA} ^{-0.23088} )$$

QC Qty	QA* Qty	Proposal Price (TY\$)	Proposal Price (FY93c\$)	regression (FY93c\$)	learn factor	rate factor	factors product	Propose vs Regress	delta	* log(QC)	* log(QA)	* log(Y)
1	0.0002			\$300,594	1.00	7.54	7.54			0.000	-3.799	
1,747	0.277	\$43,340	\$41,513	\$41,180	0.77	1.34	1.03	0.84	\$333	3.242	-0.557	4.618
1,800	0.286	\$43,015	\$41,202	\$40,854	0.77	1.34	1.02	0.84	\$348	3.255	-0.544	4.615
2,000	0.317	\$41,889	\$40,124	\$39,724	0.76	1.30	1.00	1.04	\$400	3.301	-0.498	4.603
2,500	0.397	\$39,600	\$37,931	\$37,433	0.76	1.24	0.94	1.34	\$499	3.398	-0.401	4.579
3,000	0.476	\$37,824	\$36,230	\$35,659	0.75	1.19	0.89	1.64	\$571	3.477	-0.322	4.559
3,500	0.556	\$36,384	\$34,851	\$34,225	0.75	1.15	0.86	1.84	\$626	3.544	-0.255	4.542
4,000	0.635	\$35,181	\$33,698	\$33,029	0.75	1.11	0.83	2.04	\$689	3.602	-0.197	4.528
4,500	0.714	\$34,153	\$32,714	\$32,010	0.74	1.08	0.80	2.24	\$704	3.653	-0.146	4.515
5,000	0.794	\$33,259	\$31,857	\$31,124	0.74	1.05	0.78	2.34	\$733	3.699	-0.100	4.503
5,500	0.873	\$32,471	\$31,102	\$30,344	0.74	1.03	0.76	2.44	\$758	3.740	-0.059	4.493
5,976	0.949	\$31,799	\$30,459	\$29,681	0.74	1.01	0.74	2.64	\$778	3.776	-0.023	4.484
7,221	0.198	\$43,498	\$40,770	\$42,349	0.73	1.45	1.06	-3.94(\$1,578)	*	3.859	-0.704	4.610
7,476	0.238	\$41,639	\$39,028	\$40,516	0.73	1.39	1.02	-3.84(\$1,488)	*	3.874	-0.623	4.591
7,976	0.317	\$38,923	\$36,482	\$37,826	0.73	1.30	0.95	-3.74(\$1,343)	*	3.902	-0.498	4.562
8,476	0.397	\$36,939	\$34,623	\$35,849	0.73	1.24	0.90	-3.54(\$1,227)	*	3.928	-0.401	4.539
8,976	0.476	\$35,393	\$33,174	\$34,302	0.72	1.19	0.86	-3.44(\$1,128)	*	3.953	-0.322	4.521
9,476	0.556	\$34,137	\$31,996	\$33,039	0.72	1.15	0.83	-3.34(\$1,043)	*	3.977	-0.255	4.505
9,976	0.635	\$33,084	\$31,009	\$31,978	0.72	1.11	0.80	-3.14 (\$959)	*	3.999	-0.197	4.491
10,476	0.714	\$32,183	\$30,165	\$31,066	0.72	1.08	0.78	-3.04 (\$901)	*	4.020	-0.146	4.480
10,976	0.794	\$31,398	\$29,429	\$30,270	0.72	1.05	0.76	-2.94 (\$841)	*	4.040	-0.100	4.469
11,476	0.873	\$30,704	\$28,779	\$29,564	0.72	1.03	0.74	-2.74 (\$786)	*	4.060	-0.059	4.459
11,976	0.952	\$30,084	\$28,198	\$28,933	0.72	1.01	0.73	-2.64 (\$735)	*	4.078	-0.021	4.450
12,276	1.000	\$29,742	\$27,877	\$28,584	0.72	1.00	0.72	-2.54 (\$707)	*	4.089	0.000	4.445
13,521	0.198	\$44,886	\$41,150	\$41,419	0.71	1.45	1.04	-0.74 (\$269)	*	4.131	-0.704	4.614
13,776	0.238	\$42,967	\$39,390	\$39,649	0.71	1.39	0.99	-0.74 (\$259)	*	4.139	-0.623	4.595
14,276	0.317	\$40,164	\$36,821	\$37,054	0.71	1.30	0.93	-0.64 (\$234)	*	4.155	-0.498	4.566
14,776	0.397	\$38,117	\$34,944	\$35,151	0.71	1.24	0.88	-0.64 (\$207)	*	4.170	-0.401	4.543
15,276	0.476	\$36,522	\$33,482	\$33,662	0.71	1.19	0.84	-0.54 (\$181)	*	4.184	-0.322	4.525
15,776	0.556	\$35,225	\$32,293	\$32,449	0.71	1.15	0.81	-0.54 (\$156)	*	4.198	-0.255	4.509

16,276	0.635	\$34,140	\$31,298	\$31,429	0.71	1.11	0.79	-0.44	(\$131) *	4.212	-0.197	4.496
16,776	0.714	\$33,210	\$30,446	\$30,553	0.71	1.08	0.77	-0.44	(\$107) *	4.225	-0.146	4.484
17,276	0.794	\$32,399	\$29,702	\$29,788	0.71	1.05	0.75	-0.34	(\$86) *	4.237	-0.100	4.473
17,776	0.873	\$31,683	\$29,046	\$29,110	0.71	1.03	0.73	-0.24	(\$64) *	4.250	-0.059	4.463
18,276	0.952	\$31,044	\$28,460	\$28,503	0.71	1.01	0.71	-0.24	(\$45) *	4.262	-0.021	4.454
19,521	0.198	\$46,959	\$42,119	\$40,884	0.70	1.45	1.02	2.94	\$1,235 *	4.291	-0.704	4.624
19,776	0.238	\$44,952	\$40,319	\$39,145	0.70	1.39	0.98	2.94	\$1,175 *	4.295	-0.623	4.606
20,276	0.317	\$42,020	\$37,689	\$36,597	0.70	1.30	0.92	2.94	\$1,092 *	4.307	-0.494	4.576
20,776	0.397	\$39,878	\$35,768	\$34,730	0.70	1.24	0.87	2.94	\$1,039 *	4.318	-0.401	4.553
21,276	0.476	\$38,209	\$34,271	\$33,270	0.70	1.19	0.83	2.94	\$1,001 *	4.328	-0.322	4.535
21,776	0.556	\$36,853	\$33,055	\$32,080	0.70	1.15	0.80	2.94	\$975 *	4.338	-0.255	4.519
22,276	0.635	\$35,717	\$32,036	\$31,082	0.70	1.11	0.78	3.04	\$954 *	4.348	-0.197	4.506
22,390	0.653	\$35,482	\$31,825	\$30,875	0.70	1.10	0.77	3.04	\$950 *	4.350	-0.185	4.503

\* QA normalized by dividing by the maximum economic rate, 6,300.

HELLFIRE II  
MAXIMUM YEARLY QUANTITY

year	annual qty	X cum qty	propose unit (Esc\$)	propose unit cost (FY93c\$)	lot cost	Y cum average price (FY93c\$)	regress (FY93c\$)	delta	*	log(X)	log(Y)
		1					\$39,694		*	0.000	
FY93	5,976	5,976	\$31,799	\$30,459	\$182,021,882	\$30,459	\$30,221	\$237	*	3.776	4.484
FY94	6,300	12,276	\$29,742	\$27,877	\$175,825,269	\$29,134	\$29,547	(\$413)	*	4.089	4.484
FY95	6,000	18,276	\$31,044	\$28,460	\$170,759,076	\$28,913	\$29,181	(\$268)	*	4.262	4.461
FY96	4,114	22,390	\$35,482	\$31,825	\$130,929,185	\$29,448	\$28,995	\$452	*	4.350	4.469

## Regression Output:

Constant	4.598722	\$39,694
Std Err of Y Est	0.007422	
R Squared	0.631481	
No. of Observations	4	
Degrees of Freedom	2	

X Coefficient(s)	-0.031354353	slope	97.85%
Std Err of Coef.	0.0169368148		

HELLFIRE II  
MINIMUM YEARLY QUANTITY

year	annual qty	X cum qty	propose unit (Esc\$)	propose unit cost (FY93c\$)	lot cost	Y cum average price (FY93c\$)	regress (FY93c\$)	delta	*	log(X)	log(Y)
		1					\$42,492		*	0.000	
FY93	1,747	1,747	\$43,340	\$41,513	\$72,523,927	\$41,513	\$41,417	\$96	*	3.242	4.618
FY94	1,245	2,992	43,498	\$40,770	\$50,759,218	\$41,204	\$41,341	(\$137)	*	3.476	4.618
FY95	1,245	4,237	44,886	\$41,150	\$51,231,271	\$41,188	\$41,292	(\$103)	*	3.627	4.618
FY96	1,245	5,482	46,959	\$42,119	\$52,438,743	\$41,400	\$41,255	\$145	*	3.739	4.617

## Regression Output:

Constant	4.628309	\$42,492
Std Err of Y Est	0.001813	
R Squared	0.198725	
No. of Observations	4	
Degrees of Freedom	2	

X Coefficient(s)	-0.003431930	slope	99.76%
Std Err of Coef.	0.0048728927		

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HELLFIRE II

YEARLY PRODUCTION QUANTITY	R squared
2,000	0.92
4,000	0.90
maximum	0.63
minimum	0.20
2,000 in odd years and 4,000 in even years	0.88
maximum in odd years and minimum in even years	0.00

## **Learning and Production Rate in Cost Estimating**

### **Theme Topic**

**Innovative Estimating Techniques for Business Base Changes & Related Overhead Impacts**

### **Abstract**

Production rate is a very important factor in estimating manufacturing costs. Actual price data shows that production rate has a much greater affect on cost than learning. Hellfire missile price data shows that the production rate slope is 85% and the learning slope is 98%.

Cost analysts frequently use learning as the only variable in creating cost estimating relationships from historical data bases for manufacturing costs. Predicting future costs due to changes in annual and cumulative production quantities should use both learning and production rate to prevent erroneous cost estimates. Large errors in predicting cost and quantities can result due to ignoring rate.

When production rate is considerable less than Maximum Economic Rate, which is frequent in U S Army weapon systems production, then production rate should be a factor in manufacturing cost estimates. High production rate lowers unit cost, and vice versa low production rate increases unit production cost. By quantifying the production rate factor and learning factor, we can spend the taxpayer's money more wisely and plan smarter procurement strategies.

### **How Subject Relates to Theme**

Lower production rate is a frequent reaction to cutting current costs, but this usually results in a higher unit production cost due to the government and contractor's fixed costs or business base. Production rate is shown to have a much greater impact than learning in manufacturing cost estimating relationships and unit production costs.

Alan G. Markell  
Operation Research Analyst  
U S Army, Missile Command  
Air-to-Ground Missile Systems Project Office  
SFAE-MSL-HD-M-E  
Redstone Arsenal, AL 35898-5610  
commercial (205) 876-9437  
DSN 746-9437

#### SHORT PERSONAL BIOGRAPHY - Alan Markell

Alan has a Bachelor of science degree with a major in mathematics and a minor in physics from the University of Southern Mississippi, a master of science degree in geophysics from the University of Houston, a certificate in petroleum land management from the Oklahoma City Community College. Alan worked 15 years in oil and gas exploration prior to beginning work for the U S Army.

Alan began his Operations Research Analyst career 5 and one half years ago. Alan worked in the Directorate for Systems and Cost Analysis for three years at the U S Army Aviation Systems Command (AVSCOM) now called ATCOM in Saint Louis, Missouri (from 1/88 through 3/91).

Alan has worked in the Cost Analysis Branch of Air-to-Ground Missile Systems Project Office at Redstone Arsenal Alabama for the last 2.5 years (from 3/91 though the present 9/93).

Alan has a wife and two young children.



Alan Markell  
11,410 Woodcrest Drive SE  
Huntsville, Alabama 35803  
home (205) 881-5790  
office (205) 876-9437

**PERSONAL:**

age: 46                      excellent health  
height: 5' 8"                married, 2 children  
weight: 170

**EDUCATION:**

B S 1971      University of Southern Mississippi  
              major: mathematics      minor: physics  
  
M S 1977      University of Houston  
              major: geophysics  
  
C M 1982      Oklahoma City Community College  
              major: petroleum land management

**EXPERIENCE:**

9/86-present    U S Department of Defense, operations research  
                  analyst  
  
10/83- 9/86     Markell Oil Company, Oklahoma City, OK  
                  Geophysical consulting. Economic evaluation of  
                  "distress sale" oil and gas wells and recommend  
                  bid, negotiate joint ventures, try to collect  
                  accounts receivable.  
  
5/77-10/83      oil companies, Oklahoma City, OK  
                  Geophysical interpretation with recommendation,  
                  such as, lease land, drill, do not drill, farm  
                  out, acquire more information, plan and  
                  recommend geophysical petroleum exploration  
                  programs, solicit bids for exploration  
                  programs, quality control seismic acquisition,  
                  quality control seismic and gravity computer  
                  processing, determine availability, sources,  
                  and quality of geophysical data.  
  
9/71-12/75      Seismograph Service Corp, Tulsa, OK  
                  Supervisor of scientific computer data  
                  processing center (5 men), party chief of  
                  seismic crew (25 men), supervisor of two  
                  seismic crews (50 men)